Stable and Efficient Magnetic Field Optimization of 18 MeV Sector Focused Cyclotron Magnet Using Bezier Curve Fitting

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A cyclotron magnet with a reference magnetic field of 18 MeV was designed and optimized for the production of ^{18}F radioisotope for positron emission tomography (PET) employing the Bezier curve, which was adopted as an interpolating method for smoothing the optimal shape and decreasing the computing time significantly. The design of the cyclotron magnet includes the process of designing a magnetic field for the stable acceleration of the particles to the desired energy level. The magnetic field creates a stable orbit while the particles are accelerating, allowing it to focus the bunch particles. Therefore, the electromagnet should generate a change of the magnetic field in the azimuthal direction to induce vertical and horizontal focusing. Accordingly, four sectors providing hills and valleys were selected. The magnetic field is designed by changing the angle of the hill along the radius; further, the magnetic field is sensitive to small changes of this variable. Therefore, expertise and proficiency are required to design the optimum magnetic field. In addition, much repetition is required owing to the shape sensitivity of the magnetic field. A systematic numerical approach is needed to minimize this repetition and to increase the accuracy of the magnetic field. Accordingly, we adopted the sequential approximation technique incorporated with the Bezier curve fitting and controlled the shape of the curve using design variables. Finally, through the extended paper, the accuracy and feasibility of optimized magnetic field will be tested by the beam simulation.

***Index Terms*—Cyclotron magnet, Bezier curve, Optimization, Sequential approximation technique, Isochronous condition**

I. Introduction

The design of a cyclotron magnet includes the process of designing a magnetic field to accelerate the particles to the desired energy level, which requires expertise and proficiency. The process of determining the optimal design of the magnetic field is time-consuming. Therefore, this study proposes an optimum design of the cyclotron magnet by applying Bezier curve, which can decrease the computing time and improve the accuracy of the magnetic field. The proposed cyclotron magnet is an apparatus for the production of ^{18}F radioisotope for positron emission tomography (PET) used to diagnose cancer. An energy level of 18 MeV or more is not necessary for the production of ^{18}F radioisotope. Therefore, the energy level of 18 MeV was selected considering the production efficiency and energy margin [1]. Further, we applied a sector composed of a hill and a valley in the electromagnet for the stable acceleration of the particles and caused a change of magnetic field in the azimuthal direction, thereby satisfying the isochronous condition by relativity.

The particles should be accelerated in a stable magnetic field to achieve the desired energy level; therefore, optimum design of the electromagnet is essential. The angle of the hill along the radius, which is a design variable, results in an irregular shape, which can derive a magnetic field that is unacceptable. In order to solve this problem, the Bezier curves can be used in designing electromagnets to obtain a smooth shape and a stable magnetic field. The Bezier curves are commonly used in geometrically sensitive branches that affect magnetic fields, flow, etc. [2]–[3]. The control points that control the shape of the Bezier curve and the parts that directly affect the magnetic field are set as design variables and the optimization process is performed through the sequential approximation technique.

II. Basic Model and Magnetic Field Design of 18 MeV Cyclotron Magnet

To design the basic model of the cyclotron magnet, the main parameters such as size and initial magnetic field have to be determined. Further, considering the type of accelerated particles and the maximum acceleration energy, the magnetic rigidity can be determined as

$$B \cdot r = \sqrt{T^2 + 2TE_0} \,/\, (300 \cdot Z) \tag{1}$$

where T is the maximum acceleration energy of particles, E_0 is the rest energy of a proton, and Z is the mass number. The calculated magnetic rigidity is 0.616 T·m; furthermore, the parameters of the basic model are determined through the Lorentz factor (1.019) and the initial magnetic field is determined by the rotational angular frequency of particle (439.8 MHz).

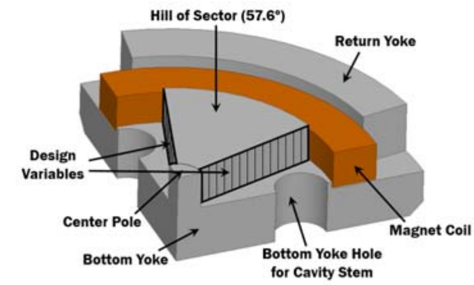

Fig. 1. Initial model of 18 MeV cyclotron magnet (1/8 model)

The initial model is shown in Fig. 1. It is necessary to change the magnetic field according to the azimuthal direction in order to accelerate the particles steadily. Therefore, vertical and horizontal focusing of the particles is induced by applying sectors composed of hills and valleys. After determining the initial model of the cyclotron magnet, it should be adjusted to the isochronous condition for a steady acceleration of the particles [4]. The isochronous field is expressed in terms of the radius of the electromagnet as

$$B_{iso} = \frac{B_0}{[1-(cqB_0 r / E_0)^2]} \quad (2)$$

where B_0 is the initial magnetic field, c is the speed of light, q is the quantity of electric charge, and r is the radius of the electromagnet. The magnetic field of the initial model should be designed to match the isochronous field. Using the magnetic field corresponding to the isochronous field, the magnetic field difference owing to the difference between the RF and particle rotation frequency is expressed as

$$\Delta B(r) = \Upsilon^2 B(r) \Delta f(r) / f_p(r) \quad (3)$$

where γ is the Lorentz factor, $B(r)$ is the magnetic field as a function of the radius, $f_p(r)$ is the frequency of rotating particles, and $\Delta f(r)$ is the difference between the RF and frequency of rotating particles [5]. The calculated magnetic field difference and the initial magnetic field design can be used to determine the reference magnetic field of 18 MeV.

III. Sequential Approximation Technique with Bezier Curve for Magnetic Field Optimization

3D magnetic field analysis of a cyclotron magnet requires much time and the distribution of the magnetic field is sensitive to even small changes in shape. Therefore, there are time constraints in interpreting and verifying the shapes within the design variable ranges. Hence, the optimization process is essential and it is important to select the appropriate design variables. In order to obtain a uniform magnetic field distribution, the Bezier curves, which hold the advantages of flexibility and continuity, are expressed as

$$R(u) = \sum_{i=0}^{n} B_{n,i}(u) V_i \quad (4)$$

where V_i is the control point, $B_{n,i}(u)$ is the Bezier coefficient, u is the independent variable of the Bezier curve, and the number of control points is n+1. A large number of adjustment points are required to improve the flexibility of the design, resulting in a high order polynomial. As shown in Fig. 2, the control point that determines the shape of the Bezier curve is used as the design point.

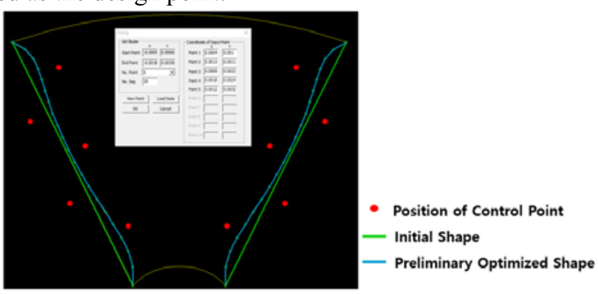

Fig. 2. Control points of magnetic field optimization using Bezier curve

Since the distribution of the initial magnetic field is important for the acceleration of the particles, the central shape of the electromagnet has more influence on the design than the outside of the electromagnet. Therefore, the variable range of the control point close to the center of the electromagnet is small, and has to be designed to gradually increase toward the outside. The Latin Hypercube Sampling technique, which is an experimental design method, was used to determine the magnetic field distribution according to the position of the initial control point. Through the finite element method (FEM) using the design points, the correlation with the objective function was analyzed. Based on the analytical data, an approximate model was constructed using the Kriging technique. This approximated model replaced the FEM analysis, which was performed later. The optimal design variables were extracted using the restricted evolution strategy, which is an optimization algorithm, because the approximated model does not take much time to obtain the results of the input values. Subsequently, the FEM analysis is performed on the extracted optimum design variables and the convergence is confirmed. If convergence does not occur, the process returns to the Kriging stage. This process is repeated until the convergence is confirmed [6].

IV. Result and Discussion

Figure 3 shows the isochronous field for obtaining 18 MeV energy, the magnetic field of the initial model, and the reference magnetic field corrected for the difference between the RF and particle rotation frequencies. The magnetic field of the initial model was optimized for the reference field using Bezier curves.

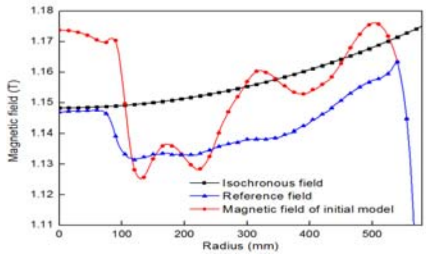

Fig. 3. Comparison of calculated average fields according to radius in initial design process

Finally, the optimized magnetic field uses beam simulation to calculate the phase error, vertical and horizontal tune of the particles, and particle trajectory. Therefore, through the extended paper, we will verify the accuracy and validity of the designed magnetic field via the optimization process using the Bezier curve.

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